Time Accuracy Verification by a Cooling Decay Test

For any dynamic simulation it is important to verify the time accuracy of the procedure. In this case we consider a cooling example which yields a simplified analytical solution for comparison.

A 20m cube (20m x 20m x 20m) has an initial internal air temperature of 50^{o}C throughout its volume. The boundary surfaces of the cube are held at a constant temperature of 21^{o}C. The average internal temperature should follow an exponential decay curve.

First we shall derive a simplified analytical solution to this problem. If we assume an average heat transfer coefficient of 3 W/m^{2}K then the energy leaving the room in a time of Δt can be written as

$$(Tr - 21) \times 2400 \times 3 \times \Delta t$$

where Tr is the average room temperature and the internal surface area of the cube is 2400m^2. This must be balanced by a corresponding loss of internal energy leading to a drop in the average internal temperature given by ΔT which can be written as follows

$$-9600 \times 1005.7 \times \Delta T$$

where 9600kg is the mass of air in the cube and 1005.7J/kg^{o}C is the heat capacity of air. Putting these two relations together and allowing $\Delta \rightarrow 0$ gives the differential equation.

$$\frac{dT}{dt} = -(Tr - 21) \times 7.46e - 4$$

Separating variables and integrating gives,

$$\int \frac{dT}{(Tr - 21)} = -7.46e - 4 \times t$$

which can be solved to obtain,

$$\ln\left[K \times (Tr - 21)\right] = -7.46e - 4 \times t$$

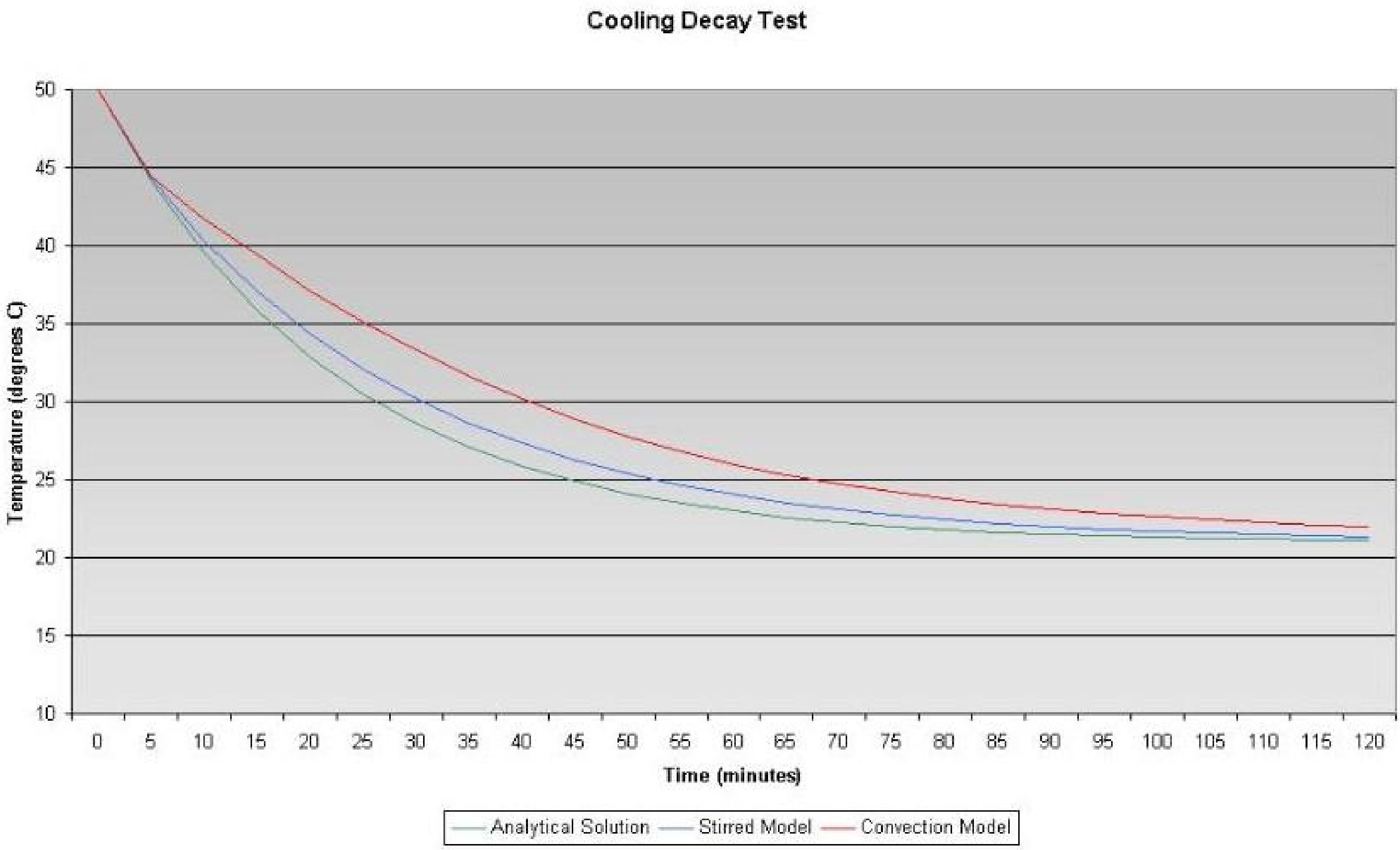
where K is the constant of integration. We know that at the start of the run t=0, the temperature is 50^{o}C so using this information gives a value K=1/29, so the final solution is

$$T = 29 \times e^{-7.47e-4 \times t} + 21 .$$

We constructed two CFD models with the boundary conditions and geometry described above. Each computational cell is one cubic metre giving a mesh size of 20x20x20. The first included a large ‘fan’ to counter the stratification effects and stir the air in the room, the second was allowed to convect naturally from buoyancy forces.

The simplified analytical solution we developed was based on an average air temperature. It represents the situation where the air in the volume is perfectly, or completely, mixed. To try and mimic this situation as closely as possible the first CFD model has a horizontal rectangle positioned just above the centre of the space with a prescribed downward velocity. The air is pushed downward, against the buoyancy forces, through this rectangular section and then flows back up either side to the top where it is pulled down and through the middle of the space once more. The second CFD model was constructed to show the more realistic situation where it is the buoyancy forces alone that lead to convection currents, we expect to see more stratification in this model so the decay should not be as fast as the analytical or stirred solution.

Comparing Results



Conclusion

The graph shows the analytical solution compared to the average air temperature in each of the CFD runs at five minute intervals.

As expected the simplified analytical solution decays the fastest as this represents the temperature variation through time for a perfectly stirred volume i.e. where the average temperature interacts directly with the enclosing surfaces. The more realistically and less perfectly stirred simulation including the fan follows very closely but with a slightly slower decay due to the temperature variations being modelled more accurately across the volume. Finally, the freely convecting example, being the least stirred of all with the largest temperature variations, is the slowest to decay. All the curves have the same time characteristic response for the air mass, showing the accuracy of the numerical dynamic modelling used in the simulations.

The fan CFD model took approximately thirty-five minutes to calculate the results for the two hours of simulated time for the comparison, while the naturally convecting model took only 30 seconds for the same simulated period of two hours. This is due to the algorithm automatically choosing the optimum time step for each situation. In the case of the fan model there is considerably more convective activity which, to capture accurately, requires a smaller time step and consequently a lot more computation meaning a longer run time. The naturally convecting model had much smaller convective fluxes and so the algorithm can choose a longer time step while still ensuring time accuracy. So, whether taking a large number of small steps or, a small number of large steps, the decay profile remains the same showing the consistency and accuracy of the dynamic modelling algorithm once more. This is similar to the principle of ‘grid convergence’ where the mesh density only needs to be refined to the point where the patterns of flow have been captured by the simulation, but in this case the resolution is time based rather than spatial.